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Piezoelectric Transformer

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Recent applications of piezoelectric materials to the active control of structural systems have been approached as a control problem in which actuator and sensor dynamics are important. Although successful control has been achieved, the synthesis of smart structures is difficult without a unified formulation. This work describes a different modeling and design method whereby the piezoelectric system is formulated by two sets of second-order equations, one for the mechanical system, and the other for the electrical system, coupled through the piezoelectric effect. The solution to this electromechanical coupled system gives a physical interpretation of the piezoelectric effect as a piezoelectric transformer. The piezoelectric transformer is a part of the piezoelectric system, which transfers the applied mechanical force into a force-controlled current source, and short circuit mechanical compliance into capacitance. It also transfers the voltage source into a voltage-controlled relative velocity input and free motional capacitance into mechanical compliance. The formulation and interpretation simplify the modeling of smart structures and lead to physical insight that aids the designer. Because of its physical realization, the smart structural system can be unconditional stable and effectively control responses. This new concept has been demonstrated in three numerical examples for a simple piezoelectric system.

Nomenclature

A	= dynamic stiffness when circuit is open
B^{-1}	= clamped motional capacitance
C, c	= capacitance matrix
D, d	= damping matrix
f	= applied mechanical force
H, h	= piezoelectric constant matrix
i	= electric current
i_f	= force-controlled current source
K, k	= stiffness matrix
L	= inductance matrix
M, m	= mass matrix
P	= electric power
p	= applied collinear force pair
Q	= output weighting matrix
q	= electric charge
R	= resistance matrix or input weighting matrix
r	= mechanical displacement
T	= transformation matrix
u	= input vector
V	= process noise covariance
v	= applied electric voltage
W	= measurement noise covariance
w_1, w_2	= input vector
$\mathcal{X}_v$	= voltage-controlled velocity input
x	= state vector
y_1, y_2	= output vector
δ	= relative displacement
ζ	= damping ratio
τ	= internal collinear force pair
ϕ	= electric potential difference
ω	= natural frequencies

Superscripts

T	= transpose of a matrix or vector
-1	= inverse of a matrix
$\cdot$	= time derivative of a variable

Introduction

SECOND-ORDER controllers that act like a passive/active vibration absorber for structural systems has demonstrated great success.[1,2] A second-order controller can provide unconditional stability of the closed-loop system and physical insight. However, the applications of second-order controllers so far have been confined to the single-input/single-output, single-mode case. Researchers are still looking for multi-input/multi-output, multimode second-order controllers that can be experimentally implemented and physically interpreted. Lenning and Özgüner proposed using a second-order electrical network to simulate a structural system and treated it as a substructure.[3] Structural vibration control was implemented by assembling the substructures through sensor/actuator interfaces. Unfortunately, the electrical model they proposed required negative capacitance and inductance, which are not physically achievable. Hagood et al. have illustrated that a piezoelectric material shunted with resistors and inductors can damp vibrations of a cantilevered beam.[4,5] Use of an inductor–resistor passive electrical network to control the first bending mode was considered. In their formulation, the influence of the mechanical response to the electric potential difference was not considered. In the aforementioned studies,[2,5] each structural mode was targeted separately by electrical components. This resulted in excessively large electrical components for controlling low-frequency structural modes.[5] Chen and Lurie[6] borrowed the idea of impedance matching from electrical engineering and suggested adjusting the active member impedance to match the load impedance such that energy dissipation is maximized. To vary the active member impedance as desired, a bridge feedback loop was designed for each active member. The interpretation of the piezoelectric effect as a transformer can be found in Ref. 9. Nonetheless, only the electrical system was considered. For a piezoelectric system, coupled mechanical and electrical systems must be considered as a whole.

Inspired by previous research, I derive a unified formulation of a coupled electromechanical system in this paper. The solution to the electromechanical system gives an interpretation that the piezoelectric device contains a piezoelectric transformer, which transfers energy between mechanical and electrical systems. The electrical system in this formulation has a dynamic equation in the same form as the mechanical system. The electrical system mimics the mechanical system by matching the electrical components to their mechanical counterparts. By doing so, one can control structural dynamics by tuning electrical components or applying a voltage to the electrical system. To this end all of the structural modes are controlled simultaneously. In this paper, passive control is implemented by increasing resistance, which can be interpreted as rate feedback.

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Active control is implemented by using two methods: one is to apply voltages in the electrical system to counteract the effects of the external mechanical forces to the mechanical system; the other is to use linear quadratic Gaussian (LQG) feedback control for the electrical system to achieve disturbance rejection. In this formulation, energy is transferred between the mechanical and electrical systems through the piezoelectric transformer. A multi-input/multi-output electrical system passively or actively controlling multiple modes of a mechanical system is illustrated in three numerical examples.

Basic Formulation

A structure with piezoelectric devices can be modeled as two sets of coupled equations: the mechanical equations of motion and Maxwell's equation.[7] The former describes the equilibrium condition of mechanical forces. The latter is an electrodynamics condition of electric potential. These two sets of equations are coupled due to the piezoelectric effect. The governing equations can be written as

$$\begin{aligned} m\ddot{r} + d\dot{r} + kr - hq &= f \\ c^{-1}q - h^T r &= v \end{aligned} \tag{1}$$

The variables are the mechanical displacement, $r = \{r_i : i = 1, \dots, n\}$, and the electric charge, $q = \{q_i : i = 1, \dots, n\}$. The coefficient matrices are the mass matrix m, the damping matrix d, the stiffness matrix k, the capacitance matrix of the piezoelectric material c, and the piezoelectric constant matrix h. The forcing vectors are the applied mechanical force, $f = \{f_i : i = 1, \dots, n\}$, and the applied voltage, $v = \{v_i : i = 1, \dots, n\}$.

For the piezoelectric system shown in Fig. 1, the coefficient matrices of the equations have the following form:

$$m = \begin{bmatrix} m_1 & & & & \\ & m_2 & & & \\ & & \ddots & & \\ & & & m_{n-1} & \\ & & & & m_n \end{bmatrix}$$

$$d = \begin{bmatrix} d_1 + d_2 & -d_2 & & & \\ -d_2 & d_2 + d_3 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & d_{n-1} + d_n & -d_n \\ & & & -d_n & d_n + d_{n+1} \end{bmatrix}$$

$$k = \begin{bmatrix} k_1 + k_2 & -k_2 & & & \\ -k_2 & k_2 + k_3 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & k_{n-1} + k_n & -k_n \\ & & & -k_n & k_n + k_{n+1} \end{bmatrix} \tag{2}$$

$$h = \begin{bmatrix} h_1 & -h_2 & & & \\ & h_2 & \ddots & & \\ & & \ddots & \ddots & \\ & & & h_{n-1} & -h_n \\ & & & & h_n \end{bmatrix}$$

$$c^{-1} = \begin{bmatrix} 1/c_1 & & & & \\ & 1/c_2 & & & \\ & & \ddots & & \\ & & & 1/c_{n-1} & \\ & & & & 1/c_n \end{bmatrix}$$

Notice that the empty entity in a matrix represents zero.

In Fig. 1, the induced force $f_{\text{ind}i}$ is defined as $f_{\text{ind}i} \equiv h_i q_i - h_{i+1} q_{i+1}$, for $i = 1, \dots, n-1$, and $f_{\text{ind}n} \equiv h_n r_n$. The induced potential difference is $v_{\text{ind}1} \equiv h_1 r_1$, and $v_{\text{ind}i} \equiv h_i(r_i - r_{i-1})$, for $i = 2, \dots, n$.

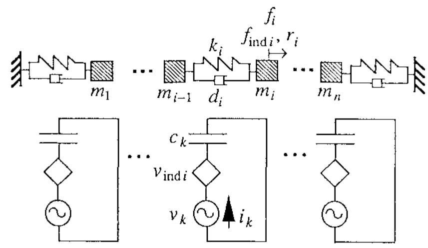

Fig. 1 Piezoelectric system described by Eq. (1).

Since a piezoelectric device is a strain-related device, one can model the system in relative displacement space rather than in absolute displacement space. Let $\delta_1 \equiv r_1$ and $\delta_i \equiv r_i - r_{i-1}, i = 2, \dots, n$, then the relative displacement, $\delta = \{\delta_i : i = 1, \dots, n\}$, is related to the absolute displacement r through the transformation matrix T by $r = T\delta$, where

$$T \equiv \begin{bmatrix} 1 & & & \\ 1 & 1 & & \\ \vdots & \vdots & \ddots & \\ 1 & 1 & \dots & 1 \end{bmatrix} \tag{3}$$

Also, the point external force f and the collinear force pair, $p = \{p_i : i = 1, \dots, n\}$, are related by $p = T^T f$. Here the force pair is defined as acting on adjacent mass blocks, having the same magnitude but in the opposite direction. When the structure has a fixed end, there is a unique p for any given f. When the structure is free-free, it requires the net applied force to be zero for p to be uniquely defined. This condition leads to the result that the rigid-body mode is not excited.

Using the preceding transformation, the governing equations become

$$\begin{aligned} M\ddot{\delta} + D\dot{\delta} + K\delta - Hq &= p \\ c^{-1}q - H^T\delta &= v \end{aligned} \tag{4}$$

where the transformed coefficient matrices are

$$M \equiv T^T m T$$

$$= \begin{bmatrix} (m_1 + \cdots + m_n) & (m_2 + \cdots + m_n) & \cdots & m_n \\ (m_2 + \cdots + m_n) & (m_2 + \cdots + m_n) & \cdots & m_n \\ \vdots & \vdots & \ddots & \vdots \\ m_n & m_n & \cdots & m_n \end{bmatrix}$$

$$D \equiv T^T d T$$

$$= \begin{bmatrix} d_1 & & \\ & d_2 & \\ & & \ddots \\ & & & d_n \end{bmatrix} + d_{n+1} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

$$K \equiv T^T k T$$

$$= \begin{bmatrix} k_1 & & \\ & k_2 & \\ & & \ddots \\ & & & k_n \end{bmatrix} + k_{n+1} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

$$H \equiv T^T h = \begin{bmatrix} h_1 & & & \\ & h_2 & & \\ & & \ddots & \\ & & & h_n \end{bmatrix}$$

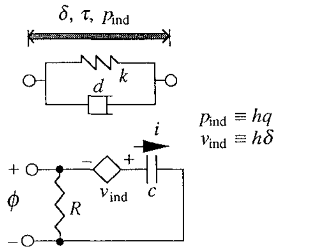

Fig. 2 Piezoelectric component.

Notice that the matrix of piezoelectric constants is diagonal in the relative displacement space but bidiagonal in the absolute displacement space. For a fixed-free system, $d_{n+1} = 0$ and $k_{n+1} = 0$. For a free-free system, d_1, k_1, and h_1 are also set to zero. When the system is free-free, the matrices $\boldsymbol{D}$, $\boldsymbol{K}$, and $\boldsymbol{h}$ are singular. In this situation, the first row and column of the system matrices are removed and form a $2 \times (n-1)$ degrees of freedom (DOF) system. The rigid-body mode is excluded from this system.

Equation (4) indicates that the applied mechanical force pair $\boldsymbol{p}$ excites the mechanical system as well as the electrical system. Also, the applied electric voltage $\boldsymbol{v}$ excites both the electrical and mechanical systems.

After the relative displacement and electric charge are determined from Eq. (4), the internal force pair τ and the electric potential difference ϕ can be obtained from the constitutive equations for a piezoelectric material. The constitutive equations for a piezoelectric material expressed in the relative displacement space are given by

$$\begin{aligned} \tau &= k\delta - hq \\ \phi &= -h\delta + c^{-1}q \end{aligned} \tag{5}$$

where all of the variables are scalar. The piezoelectric constant h has a unit of force/charge or voltage/length. This two units are identical because that the voltage is defined as work/charge.

The constitutive equations suggest a model for the piezoelectric component as shown in Fig. 2. A piezoelectric component has mechanical and electrical properties. Therefore a piezoelectric component should contain stiffness, damping, internal capacitance, and internal resistance. A voltage, $v_{ind} = h\delta$, is induced due to mechanical deformation, and a mechanical force pair, $p_{ind} \equiv hq$, is induced by charge flow. The directions of the induced voltage and the induced force pair are indicated in Fig. 2 and are matched with the sign convention used in Eq. (5).

Normally, for a piezoelectric material the internal damping is small but the internal resistance is very high. For a very high internal resistance, the circuit can be considered to be open, and thus the resistor R can be ignored in the circuit illustrated in Fig. 2.

Since a piezoelectric material can transfer energy between mechanical and electrical systems, one would expect a resistor to damp mechanical vibrations through the piezoelectric effect. An electronic passive damper is formed by connecting a resistor to a piezoelectric component. The resistor would be sized to dissipate a large amount of energy at the structural resonant frequencies. To tune the frequencies of the electrical system, inductors and capacitors are introduced into the circuit. External inductors, resistors, and capacitors, together with internal capacitors and induced voltages for the piezoelectric components, constitute a coupled electromechanical system described by

$$\begin{aligned} \boldsymbol{M}\ddot{\boldsymbol{\delta}} + \boldsymbol{D}\dot{\boldsymbol{\delta}} + \boldsymbol{K}\boldsymbol{\delta} - \boldsymbol{H}\boldsymbol{q} &= \boldsymbol{p} \\ \boldsymbol{L}\ddot{\boldsymbol{q}} + \boldsymbol{R}\dot{\boldsymbol{q}} + \boldsymbol{C}^{-1}\boldsymbol{q} - \boldsymbol{H}^T\boldsymbol{\delta} &= \boldsymbol{v} \end{aligned} \tag{6}$$

The mechanical system and electrical system equations have analogous forms. They are coupled through the piezoelectric effect.

To match the "dynamic stiffness" of the electrical system to that of the mechanical system, one replaces the values of m by that of L, d by R, and k by $(1/C)$ in Eq. (4). The electric circuit that has the same dynamics as the mechanical systems described in Eq. (6) for a fixed-fixed system together with the mechanical system are given in Fig. 3. For a fixed-free system, one simply removes R_{n+1}, and C_{n+1} from the circuit. For a free-free system, one also removes L_1, R_1, C_1, and v_1.

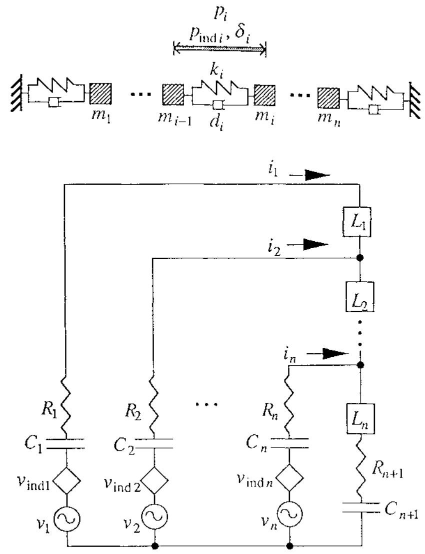

Fig. 3 Piezoelectric system described by Eq. (6).

By defining a new state variable $\boldsymbol{z} \equiv [\boldsymbol{\delta}^T \boldsymbol{q}^T]^T$ and a new forcing vector $\boldsymbol{u} \equiv [\boldsymbol{p}^T \boldsymbol{v}^T]^T$, the augmented electromechanical system can be written as

$$\mathcal{M}\ddot{\boldsymbol{z}} + \mathcal{D}\dot{\boldsymbol{z}} + \mathcal{K}\boldsymbol{z} = \boldsymbol{u} \tag{7}$$

in which

$$\mathcal{M} \equiv \begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{L} \end{bmatrix}, \qquad \mathcal{D} \equiv \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R} \end{bmatrix}, \qquad \mathcal{K} \equiv \begin{bmatrix} \boldsymbol{K} & -\boldsymbol{H} \\ -\boldsymbol{H}^T & \boldsymbol{C}^{-1} \end{bmatrix}$$

The electromechanical system described in Eq. (6) is an open-loop system. A linear, physical open-loop system is always stable because the system (7) is positive real. The quantity $\frac{1}{2}\dot{\boldsymbol{\delta}}^T\boldsymbol{M}\dot{\boldsymbol{\delta}}$ can be identified as kinetic energy, $\frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{L}\dot{\boldsymbol{q}}$ magnetic energy, and $\frac{1}{2}\boldsymbol{z}^T\mathcal{K}\boldsymbol{z}$ potential energy, in which $\frac{1}{2}\boldsymbol{\delta}^T\boldsymbol{K}\boldsymbol{\delta}$ is elastic energy, $\boldsymbol{\delta}^T\boldsymbol{H}\boldsymbol{q}$ coupling energy, and $\frac{1}{2}\boldsymbol{q}^T\boldsymbol{C}^{-1}\boldsymbol{q}$ electric energy. The virtual work done by the damping forces and the electric potential differences between two ends of resistors are $\Delta\boldsymbol{\delta}^T\boldsymbol{D}\dot{\boldsymbol{\delta}}$ and $\Delta\boldsymbol{q}^T\boldsymbol{R}\dot{\boldsymbol{q}}$, respectively. The virtual work done by external loads is $\Delta\boldsymbol{\delta}^T\boldsymbol{p} + \Delta\boldsymbol{q}^T\boldsymbol{v}$. A variable proceeded by Δ indicates a virtual quantity. Equation (7) can also be derived using Hamilton's principle and the work and the energy quantities defined earlier.

In the s domain, Eq. (7) can be written as

$$\begin{bmatrix} \boldsymbol{A} & -\boldsymbol{H} \\ -\boldsymbol{H}^T & \boldsymbol{B} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\delta} \\ \boldsymbol{q} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{p} \\ \boldsymbol{v} \end{Bmatrix} \tag{8}$$

where $\boldsymbol{A} \equiv \boldsymbol{M}s^2 + \boldsymbol{D}s + \boldsymbol{K}$, $\boldsymbol{B} \equiv \boldsymbol{L}s^2 + \boldsymbol{R}s + \boldsymbol{C}^{-1}$, and s is the Laplace variable. When $\boldsymbol{B} = \boldsymbol{H}^T\boldsymbol{A}^{-1}\boldsymbol{H}$, the matrix in Eq. (8) is singular, or else the solution of Eq. (8) is given as

$$\begin{Bmatrix} \boldsymbol{\delta} \\ \boldsymbol{q} \end{Bmatrix} = \begin{bmatrix} (\boldsymbol{A} - \boldsymbol{H}\boldsymbol{B}^{-1}\boldsymbol{H}^T)^{-1} & \boldsymbol{A}^{-1}\boldsymbol{H}(\boldsymbol{B} - \boldsymbol{H}^T\boldsymbol{A}^{-1}\boldsymbol{H})^{-1} \\ \boldsymbol{B}^{-1}\boldsymbol{H}^T(\boldsymbol{A} - \boldsymbol{H}\boldsymbol{B}^{-1}\boldsymbol{H}^T)^{-1} & (\boldsymbol{B} - \boldsymbol{H}^T\boldsymbol{A}^{-1}\boldsymbol{H})^{-1} \end{bmatrix} \begin{Bmatrix} \boldsymbol{p} \\ \boldsymbol{v} \end{Bmatrix} \tag{9}$$

The matrix $\boldsymbol{S}_0 \equiv \boldsymbol{A}^{-1}$ is the dynamic compliance of the mechanical system when the circuit is open. The matrix $\boldsymbol{C}_0 \equiv \boldsymbol{B}^{-1}$ is called the clamped motional capacitance[9] of the electric system since it is measured when the structure is clamped. The matrix $\boldsymbol{S}_1 \equiv (\boldsymbol{A} - \boldsymbol{H}\boldsymbol{B}^{-1}\boldsymbol{H}^T)^{-1}$ can be identified as the dynamic compliance when the circuit is shorted. Similarly, the matrix $\boldsymbol{C}_1 \equiv (\boldsymbol{B} - \boldsymbol{H}^T\boldsymbol{A}^{-1}\boldsymbol{H})^{-1}$

is the free motional capacitance since it is obtained when the structure is stress free.

Using the well-known matrix inversion formula,[8] the response of charge to the force and voltage inputs is then given by

$$q = B^{-1}H^T(A - HB^{-1}H^T)^{-1}p + [B^{-1} + B^{-1}H^T(A - HB^{-1}H^T)^{-1}HB^{-1}]v \quad (10)$$

In the s domain, the current is related to the charge by $i = sq$. Equation (10) suggests the circuit given in Fig. 4, in which i_f is the force-controlled current source and is defined as

$$i_f \equiv sB^{-1}H^T(A - HB^{-1}H^T)^{-1}p \quad (11)$$

Note that all of the variables in Fig. 4 are vectors or matrices, and the matrix C_M is defined as $C_M \equiv B^{-1}H^T(A - HB^{-1}H^T)^{-1}HB^{-1}$. The total capacitance is composed of two parts, the clamped capacitance C_0, and the capacitance C_M due to the piezoelectric effect. Therefore, the load of the electrical system consists of electric load and mechanical load. The mechanical load enters the electrical system through the piezoelectric effect.

Similarly, the response of relative displacement δ to the force and voltage inputs is given by

$$\delta = [A^{-1} + A^{-1}H(B - H^TA^{-1}H)^{-1}H^TA^{-1}]p + A^{-1}H(B - H^TA^{-1}H)^{-1}v \quad (12)$$

and the voltage-controlled relative velocity input $\mathcal{X}_v$ is given by

$$\mathcal{X}_v \equiv sA^{-1}H(B - H^TA^{-1}H)^{-1}v \quad (13)$$

The mechanical system described by Eq. (12) is illustrated in Fig. 5.

In Fig. 5, the matrix S_E is defined as $S_E \equiv A^{-1}H(B - H^TA^{-1}H)^{-1}H^TA^{-1}$, and $\dot{\delta} = s\delta$ is the relative velocity. It is very important to notice that a piezoelectric system should include both the electrical system illustrated in Fig. 4 and the mechanical system described in Fig. 5.

If one defines the turns ratios $N_E \equiv B^{-1}H^T$ and $N_M \equiv A^{-1}H$, then the piezoelectric component can be considered to contain a transformer. An equivalent system that interprets the piezoelectric effect as a piezoelectric transformer is given in Fig. 6.

Note that the turns ratios N_E and N_M are frequency dependent due to the dynamics of B and A, respectively. In the system illustrated in Fig. 6, the right-hand side of the piezoelectric transformer contains only mechanical components and the left-hand side only electrical components. The piezoelectric transformer is part of the piezoelectric system. The system shown in Fig. 6 is equivalent to that given in Fig. 3.

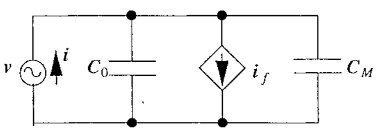

Fig. 4 Electric system described by Eq. (10).

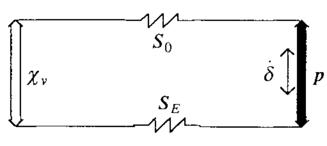

Fig. 5 Mechanical system described by Eq. (12).

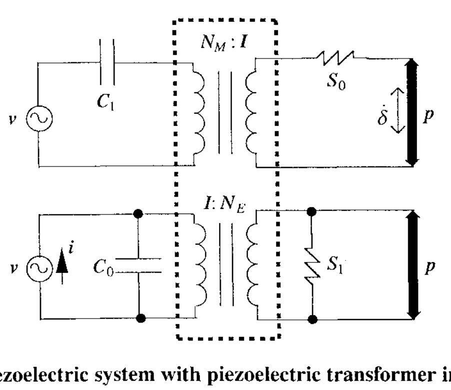

Fig. 6 Piezoelectric system with piezoelectric transformer interpretation.

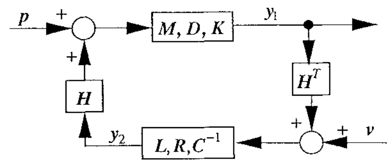

Fig. 7 Closed-loop interpretation of a piezoelectric system.

A control engineer may view the electromechanical system in a different manner. Define two open-loop systems as

System 1:

$$M\ddot{\delta} + D\dot{\delta} + K\delta = p + w_1 \qquad y_1 = \delta$$

System 2:

$$L\ddot{q} + R\dot{q} + C^{-1}q = v + w_2, \qquad y_2 = q \quad (14)$$

and design the control laws as

$$w_1 = Hy_2, \qquad w_2 = H^Ty_1 \quad (15)$$

After the loops are closed, the dynamics of the composite system are described by Eq. (6). System 1 can be considered as the system to be controlled, whereas system 2 is a dynamic compensator. The design parameters are L, R, and C^{-1}. The controller gain H is fixed by the piezoelectric material properties.

The block diagram illustrated in Fig. 7 depicts the closed-loop system. Although Eq. (6) has a closed-loop interpretation, it is still a positive, energy dissipative system. Therefore stability is guaranteed. Notice that energy is transferred between the electrical system and the mechanical system through the piezoelectric transformer.

The closed-loop interpretation of the electromechanical system gives insight to control design. To control a multimode system, the objective may be to achieve the largest energy transfer between the electrical and the mechanical systems. To have the largest energy transfer, it is common to match the "dynamic stiffness" of the electrical system to that of the mechanical system, that is, to match $Ms^2 + Ds + K$ to $Ls^2 + Rs + C^{-1}$.

Control Design

Passive Design: Energy Dissipation

For passive control, mechanical energy may be transferred into electric energy through the piezoelectric transformer and then dissipated by resistance in the electric circuit. One might expect that the larger the resistance, the more the energy will be dissipated. However, this is not the case. The power P consumed by a resistor is given by

$$P = i^2R \quad (16)$$

Equation (16) suggests that increasing either current or resistance will increase power consumption. Smaller resistance draws larger current through the resistor from both the voltage source and the force-controlled current source, whereas larger resistance draws smaller current. Therefore, resistance and current are not independent. The total power consumption P of the composite circuit described by the second-order equation in Eq. (6) can be defined as

$$P = \dot{q}^TR\dot{q} \quad (17)$$

Therefore, passive control design process reduces to determining the resistance matrix R after matching the parameters $Ms^2 + K$ and $Ls^2 + C^{-1}$.

Vibration Suppression Using the Counteracting Electric Voltage

If the external disturbance p is known, and the piezoelectric constant matrix H has full rank, and then the applied voltage to the system can be chosen such that the piezoelectric force counteracts the external mechanical force. From Eq. (9), if the active voltage is chosen to be

$$v = -BH^{-1}p = -(Ls^2 + Rs + C^{-1})H^{-1}p \tag{18}$$

then the response δ can be fully suppressed. The charge in the system, from Eq. (9), then becomes

$$q = (B - H^TA^{-1}H)^{-1}(H^TA^{-1} - BH^{-1})p \tag{19}$$

To calculate v from Eq. (18) involves time derivatives of p. Fortunately, the conditions for the piezoelectric force $HB^{-1}v$ to cancel the mechanical force p does not require dynamics in the electrical system. In other words, the inductance matrix L and resistance matrix R can be designed to be negligible small, and the first two terms on the right-hand side of Eq. (18) can be neglected.

Active Design: An LQG Approach

In the previous two approaches, only an open-loop system was considered. In this section, a controller is designed by using linear quadratic Gaussian (LQG) method. The controlled input to the system is the electric voltage, and only current output is fed back. The applied force is considered as a disturbance to the system. The control design is to choose the output weighting matrix Q, input weighting matrix R, process noise covariance V, and measurement noise covariance W such that a performance index of weighted output and input and the estimation error are minimized. The advantages of closing the feedback loop on the piezoelectric system rather than on the mechanical system is that one can control the mechanical system indirectly by controlling the coupled electrical system. In the piezoelectric system, no actuators other than piezoelectric components are involved, and measuring current is easier than measuring relative velocity. Strictly speaking, a piezoelectric component is not an actuator of the piezoelectric system but a constituent of the system.

Numerical Results

To demonstrate the concept just presented, a simple model of a 2×2 DOF composite electromechanical system is shown in Fig. 8. The structural properties are listed in Table 1.

For active control, the parameters of the circuit are chosen to match their counterparts in the mechanical system. However, for passive control, the values of the resistance are increased to dissipate more energy. The resistance in this case was chosen by trial and error until both modes were equally damped. The values of L, R, and C are given in Table 2.

Case I: Passive Design

The frequency response functions of the mechanical system (M, D, K) and the composite electromechanical system ($\mathcal{M}$, $\mathcal{D}$, $\mathcal{K}$) are illustrated in Fig. 9.

The input is the applied mechanical force pair p_2, and the output is the relative displacement δ_2. The natural frequencies and damping ratios of the two systems are given in Table 3. The composite system has two pairs of split modes compared with only two modes of the mechanical system. Because of the electrical/mechanical interaction, the damping of the composite system increases by 20 times in comparison with the mechanical system. Notice that in passive control there is no external voltages applied to the composite system.

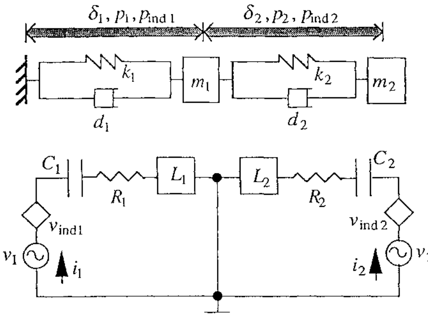

Fig. 8 2×2 DOF composite system.

Table 1 Structural properties

Element	m,kg	k,N/m	d,N-s/m	h,V/m
1	1.250	2.0e8	100.0	2.0e7
2	0.675	2.0e8	25.0	2.0e7

Table 2 Parameters of the circuit for cases I and III

Element	L, H	C, F	Case I R, Ω	Case III R, Ω
1	1.250	5.0e-9	4.0e3	100.0
2	0.675	5.0e-9	1.0e3	25.0

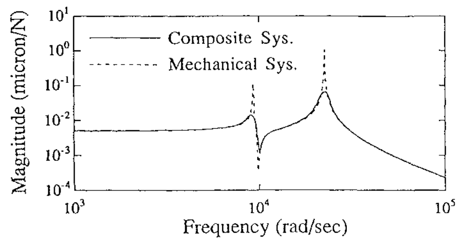

Fig. 9 Bode plot of the output δ_2 due to the input p_2.

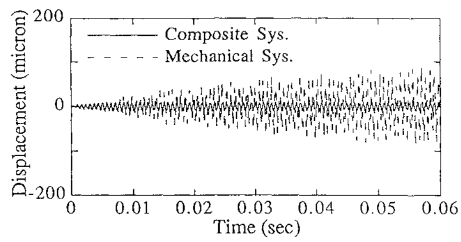

Fig. 10 Time history of r_2 (case I).

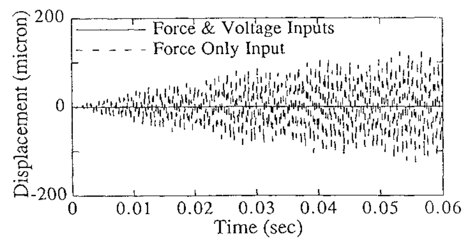

Fig. 11 Time history of r_2 (case II).

The responses of the displacement r_2 to a sinusoidal force input p for both systems are given in Fig. 10. The sinusoidal force input contains two excitation frequencies of 9.32e3 and 2.25e4 rad/s, which are the resonant frequencies of the mechanical system. It has a peak amplitude of 100 N.

Case II: Vibration Suppression Using the Counteracting Electric Voltage

In this example, the matrices H and C^{-1} are assumed to be the same as in case I, but $L = 0$ and $R = 0$. The electric voltage is simply $v = -C^{-1}H^{-1}p$. The force input is sinusoidal having frequencies of 9.27e3 and 2.24e4 rad/s, and a peak amplitude of 100 N. The response of the displacement r_2 and current i_2 are given in Figs. 11 and 12, respectively. The solid curves are the responses due to the combined effects of p and v, and the dashed curves are due to the input p only. It is clear that the counteracting force HCv prevents the external force p from exciting the mechanical system.

Case III: Active Control—A LQG Approach

The weighting matrices Q and R are chosen as $Q = 1.0e7I$, and $R = I$, where I is the identity matrix. The process noise covariance V and measurement noise covariance W are designed as $V = 5.0e7I$, and $W = I$. The natural frequencies and damping ratios of the

Table 3 Frequencies ω (rad/s) and damping ratios ζ (%)

	Mechanical System		Case I		Case II		Case III	
Mode	ω	ζ	ω	ζ	ω	ζ	ω	ζ
1	9.32e3	0.21	9.03e3	4.53	9.27e3	0.21	8.98e3	3.98
2	2.25e4	0.20	9.58e3	4.01	2.24e4	0.20	9.26e3	14.86
3	—	—	2.17e4	4.60	—	—	9.32e3	1.71
4	—	—	2.31e4	3.74	—	—	9.61e3	3.43
5	—	—	—	—	—	—	2.24e4	16.33
6	—	—	—	—	—	—	2.24e4	3.93
7	—	—	—	—	—	—	2.25e4	0.67
8	—	—	—	—	—	—	2.25e4	1.55

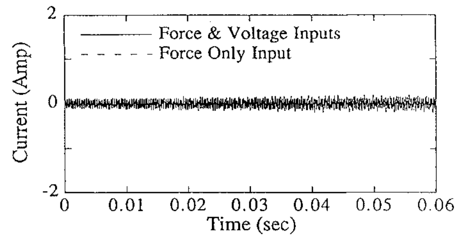

Fig. 12 Time history of i_2 (case II).

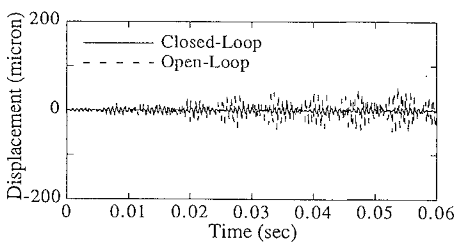

Fig. 13 Time history of r_2 (case III).

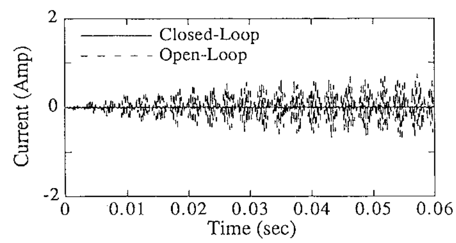

Fig. 14 Time history of i_2 (case III).

closed-loop system are given in Table 3. The closed-loop and open-loop responses of the displacement r_2 and the electric current i_2 are given in Figs. 13 and 14, respectively. The disturbance inputs are the two sinusoidal mechanical forces, p_1 and p_2. They excite the system at its open-loop natural frequencies and have peak amplitudes of 100 N. The responses of the closed-loop system show that the LQG controller can effectively reject the disturbance.

Summary

In this paper, a new concept for modeling piezoelectric components for the control of structures has been developed. A piezoelectric transformer was introduced to give a physical interpretation of the piezoelectric effect. A unified formulation of an electromechanical system was derived to passively or actively control all of the structural modes simultaneously. This approach provides physical insight for controller designs and yields a composite system that can never go unstable.The design procedure involves matching the electrical components to their mechanical analogs. The idea of using counteracting voltages to cancel external forces proposed has limitations. Yet it shows that controlled inputs play an important role in a vibration suppression problem. The LQG control example showed that structural vibration control can be implemented indirectly by controlling the coupled electrical system.

The model presented in this paper is described by a lumped-parameter system. For distributed parameter structures, the variables are spatially dependent, and therefore the lumped electrical components addressed earlier do not apply. In this case, distributed parameter electrical components are necessary to account for the spatial dependency. Continued research will be focused on this aspect.

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